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Projectile Entry Angle Determination

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ABSTRACT: Small angles of projectile entry are difficult to determine in two-dimensional targets or in three-dimensional targets where internal deflection has taken place. This results from the fact that small errors in measurement lead to large errors in the calculation of the sine function used in the estimation of the angular projection of a circle onto an inclined plane at angles of less than 20° from the normal. The use of a bullet tip scale, constructed to the dimensions of the entering projectile, allows a significantly more accurate determination of small angles of entry in targets not subject to stretching. A comparison of the relative error inherent in calculating entry angle using the sine function and measuring the angle using the bullet tip scale demonstrates the suitability of using the tip scale at angles less than 20° and the sine function at entry angles greater than 20°.

KEYWORDS: criminalistics, ballistics, scale (ratio), projectile entry, trajectory

Crime scene reconstruction in cases where a firearm has been discharged almost invariably includes the study of exterior ballistics as it applies to bullet trajectory. In most instances, the normal projectile trajectory can be ignored since the distance between the shooter and the target is usually within 9 m (10 yds) [*1*]. This results in the projectile trajectory being treated as if it were a straight line between the muzzle of the weapon and the target.

Typically, the data obtained in this type of reconstruction can include the distance between the muzzle of the weapon and the target along with an estimate of the angle of the gunbarrel relative to the target. This information can then be used to ascertain the approximate location of both the shooter and the target [*2*].

In those instances where either visible or invisible gunshot residue patterns can be detected, muzzle-to-target distance is a straightforward determination and is reasonably accurate if the same type of weapon and ammunition are used [*3*]. This is not usually the case in the determination of the relative angle between the weapon and the target which, with the exception of an undeflected shot through a three-dimensional target, is largely a matter of estimation and conjecture based on the shape of the bullet wipe, the shape of the contusion/abrasion ring at the target entry site, and the shape of the gunshot residue pattern on the target surface [*4,5*].

Theory

In theory, the determination of a projectile entry angle is contingent upon the presence of a complete bullet wipe surrounding the point of entry in the target. The shape and dimensions of this ring are the only requirements for an accurate entry angle determination in those instances where differential target surface stretching can be ignored.

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The reason for this appears to be resident in the fact that if a typical projectile is fired at the surface of a traget, it would be "seen" from the target's point-of-view as a symmetrical, two-dimensional object.

If the projectile were fired normal to the target, it would be perceived as a circle, the diameter of which would be the caliber of the bullet. If either the target or gunbarrel were at an angle other than 90° relative to each other, the projectile would be seen by the target as an ellipse. In this instance, the minor axis of the ellipse would represent the caliber of the projectile and the major axis would be indicative of the relative angle between the target and the gunbarrel (Fig. 1).

Using analytical geometry, the true angle θ can be determined from the formula:

$$\sin\theta = \frac{\text{length of minor axis}}{\text{length of major axis}}$$

Note that the angle relative to the normal to the surface is obtained by subtracting the true angle from 90°. Thus, a shot fired normal to the target (that is, 90° to the surface) is perceived by the target as approaching at 0°.

An inspection of the above formula reveals that when both axes are equal, $\theta = 90°$ and the resulting figure is a circle. As the length of the major axis increases, θ decreases and the resulting ellipse becomes more eccentric, eventually approximately a straight line of infinite length [*6*].

Unfortunately, because of the nature of the sine function, it is difficult to calculate accurately the projectile entry angle at angles between 0 to approximately 15° from the normal. This stems from the fact that small errors in measurement can lead to large errors in angle determination within this range.

Experimental Procedure

Because of the inherent inaccuracy in using the sine formula for small angles relative to the normal, it was decided to measure the small barrel-to-target angles with a bullet tip scale. This device, initially described by Mann in 1907, was originally designed to measure the effect of tipping in imbalanced bullets [*7*]. The bullet tip scale (Fig. 2) consists of a plastic overlay having dimensions which are dictated by the length and caliber of the projectile in question.

Once a scale is constructed for a particular projectile, it is superimposed on the bullet entry

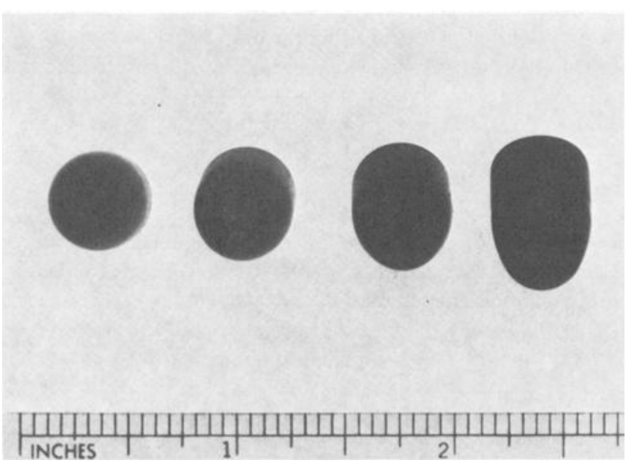

FIG. 1—*Photograph of the perceived shape of a .45 ACP projectile approaching a target at angles of 0, 15, 30, and 45°, respectively (relative to the normal). 1 in. = 25.4 mm.*

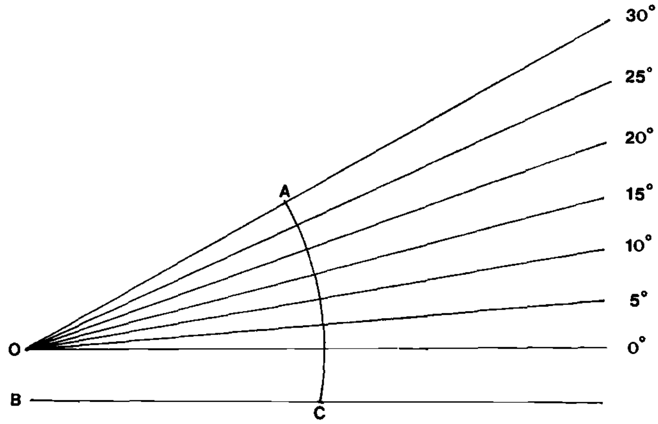

FIG. 2—*Bullet tip scale. The distance from the baseline to the 0° line* (OB) *represents one half the projectile caliber, and the arc radius over the degree scale* (OA) *represents the length of the projectile.*

hole and aligned with the arc baseline intersection (C) over the approximate point of impact. The scale is then rotated about this point such that the arc degree scale line (AC) intersects the furthest point of the bullet wipe along the long axis of the entry hole. If the projectile struck the target normally, its entry hole wipe would just touch the 0° line. At any other angle, the entry hole wipe would cut one or more of the degree lines. Figure 3 illustrates the proper orientation of the scale to a projectile entry hole.

In addition to determining the projectile entry angle, the barrel-to-target direction must also be ascertained. This is simply accomplished by treating an imaginary line between the point of impact and the furthest point of the major axis tip as a clock hand rotating about the point of impact.

To test both the calculated sine function and the estimated bullet tip scale approaches to entry angle determination, a number of test-firings (five for each caliber at each angle) were conducted using .22 LR, .32-20, .32 ACP, .380 ACP, .38 SPL, and .45 ACP caliber ammunition. These were fired at barrel to target angles of 0, 5, 10, 15, 30, and 45°, respectively, (relative to the normal) at 0.8-mm (1/32-in.) thick cardboard screens. In all cases, the targets were rigidly supported at a given angle while the weapon was mounted at a distance of 1.5 m (5 ft) in a bench vise with a spirit level attached to either the barrel or slide.

The results of the test-firings were individually calculated by the sine function or estimated using the bullet tip scale or both, and the average was computed for each approach for each caliber at each angle.

Results

An inspection of the data in Table 1 reveals that at barrel-to-target angles of up to approximately 15° from the normal, the determination of the angle using the bullet tip scale is considerably more accurate than the sine function calculation. The converse is true at barrel to target angles in excess of approximately 30° from the normal. Figure 4, which represents the graph of the average of both the measured and estimated values for all calibers at each angle, illustrates this.

Discussion

From the plot in Fig. 4, it can be seen that at an angle of approximately 20° from the normal both the sine function and the bullet tip scale values deviate equally from the true value by ap-

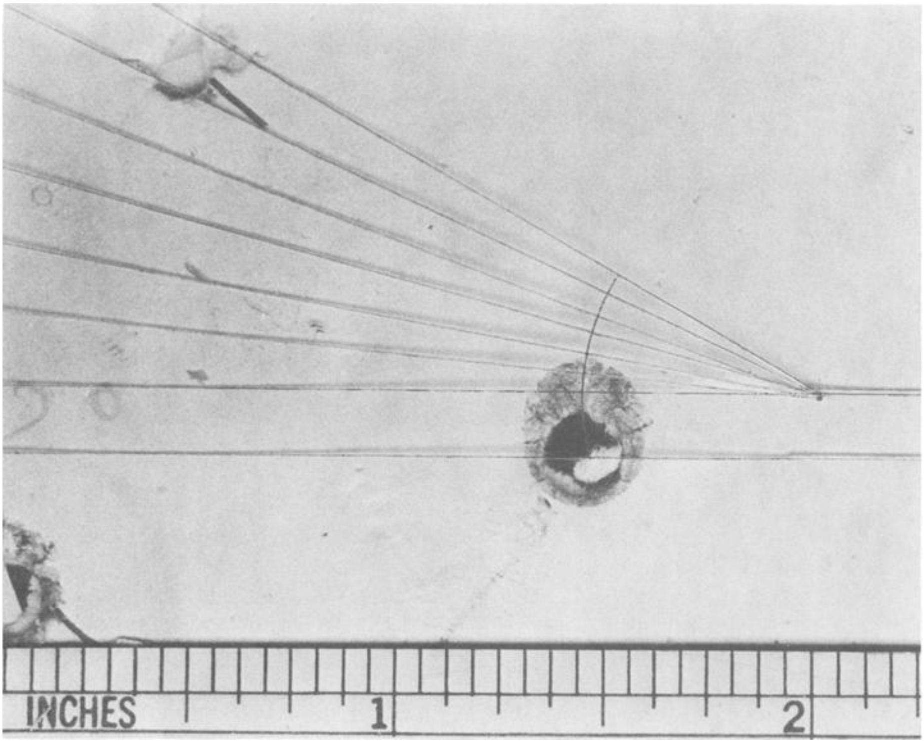

FIG. 3—*Bullet tip scale superimposed over a .32-20 entry hole of 10° (relative to the normal). This hole was produced by the projectile entering from above the target. 1 in. = 25.4 mm.*

TABLE 1—*Experimental data obtained using the sine function and estimated using the bullet tip scale. The sine function measurements were taken to the nearest 0.4 mm (1/64 in.). The bullet tip scale estimates were interpolated to the nearest 2 1/2°. The values in this table represent the average of five test-firings for each caliber at each angle.*

		Barrel-Target Angle, Degrees					
Bullet		0	5	10	15	30	45
.22 LR	tip scale	0	4	5	7	12	...
	arc sine θ	...	...	20	21	33	45
.32 ACP	tip scale	0	6	7	9	15	...
(FMJ)	arc sine θ	...	...	23	28	35	44
.32-20	tip scale	0	4	8	10	16	...
(SW)	arc sine θ	...	...	22	28	33	43
.380 ACP	tip scale	0	7	12	13	20	...
(FMJ)	arc sine θ	...	...	21	26	28	42
.38 SPL	tip scale	0	5	8	12	22	...
	arc sine θ	...	...	20	25	32	44
.45 ACP	tip scale	0	4	7	8	28	...
(FMJ)	arc sine θ	...	...	16	26	32	45
Average	tip scale	0	5	8	9	19	...
	arc sine θ	...	...	20	26	32	44
Relative	tip scale	0	0	−2	−6	−11	...
error	arc sine θ	...	...	10	11	2	−1

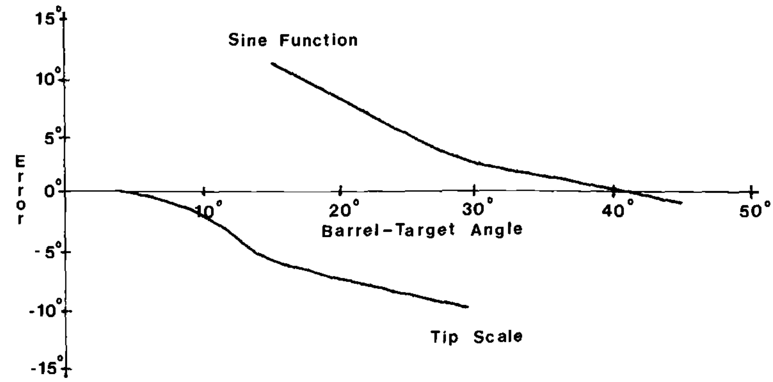

FIG. 4—*Graph of the average of all data in Table 1 illustrating the deviation of the experimental results (ordinate) from the true values (abscissa).*

proximately 8°. This suggests the use of the bullet tip scale when determining angles of less than 20° and the sine function at angles of greater than 20°.

For the most part, the choice of a particular method will be dictated by the shape of the projectile wipe. Where little or no eccentricity is noticed, the bullet tip scale is the method of choice. Where definite elongation is noted (usually in excess of 20° from the normal), the sine function calculation is to be preferred. In the situation where there exists doubt as to which method should be used (that is, approximately 20° from the normal), both the sine function and the bullet tip scale values should be measured and an average taken.

Error

A number of possible sources of error were noted during the course of the study. The two most obvious of these included small measuring errors and incorrect positioning and reading of the bullet tip scale. These were minimized by averaging three separate determinations for each shot at each angle.

Another source of error was bullet tipping. This was especially noticeable with flat-tipped lead projectiles and with jacketed hollow point ammunition. In those instances where tipping was observed (by placing a screen normal to the line of fire behind the target screen), the results were discarded.

A final source of error was associated with jacketed ammunition. In all instances, it was observed that the bullet wipe ring diameter was approximately 7% less than the true bullet caliber. This necessitated a certain degree of estimation in positioning and reading the bullet tip scale along with an adjustment in the calculation of the sine function.

Blind Trials

A limited number of blind trials were conducted to assess both approaches to projectile entry angle determination. In the trials, both experienced and inexperienced individuals were asked to measure and calculate the angle using the sine formula and to estimate the angle using the bullet tip scale.

Within the range of 0 to 15°, the average error was ±3° using the bullet tip scale. In the range of 15 to 25°, the average error was ±6° using the average of both approaches. From 25 to 45°, the average error was ±2° using the sine function.

Conclusion

This study demonstrates the efficacy of using a bullet tip scale for the determination of small projectile entry angles of up to approximately 20° from the normal. It is clear from the data that the more accurate sine function simply cannot duplicate this because of measuring error.

On the other hand, the sine function is certainly more useful in those instances where the projectile entry angle is greater than 20° from the normal. In addition, preliminary studies indicate that the sine function has application in determining the angle of projectile entry in glass[2] and in certain instances of bullet ricochet [*8*].

The obvious limitation in the use of the bullet tip scale occurs in attempting to estimate the entry angle in tissue. This results from the fact that the bullet tip scale is based on the dimensions of a particular projectile and not on the projectile wipe in tissue. Here, small angles of entry cannot be accurately determined. However, angles in excess of 20° can be approximated by applying the sine formula (if uniform tissue stretching can be assumed).

[2]P. J. Cashman, unpublished data, 1982.

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